Strong tunneling and charge quantization in S-I-N Coulomb blockade structures

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We study the charge of a small normal island connected by a tunnel junction to a superconducting lead. Unlike the N-I-N case, the steps of the Coulomb staircase remain sharp even if the conductance of the tunneling barrier exceeds e^2/h. One can observe the transition from sharp steps to the smeared ones by applying magnetic field to destroy the superconductivity.

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The charge of a small conducting island weakly coupled to an electron reservoir (lead) by a tunneling barrier shows a step-like dependence on the gate voltage due to the well-known phenomenon of Coulomb blockade.[1] It was shown in Refs. 2,3 that the quantization of charge is not precise: the processes of virtual tunneling between the lead and the grain result in smearing of charge steps. The smearing is especially dramatic in the regime of strong tunneling, when the conductance G of the tunneling barrier is large, $G \gtrsim e^2/h$. The evolution of the charge quantization with the increasing G was studied in semiconductor quantum dots, where the junction conductance can be tuned between $G = 0$ and $G \sim e^2/h$.[4–6] Charge quantization was also observed[7] in a metallic "electron box" device, yielding a sharp Coulomb staircase at small G. In a metallic device, one can achieve the tunneling junction conductance $G \gg e^2/h$, Ref. 8. In this case, theory[9–11] predicts only weak oscillations of charge, with amplitude $\sim e\exp(-hG/2e^2)$. However, the layout of a metallic device does not allow one to vary the junction conductance, and the crossover between weak and strong tunneling cannot be observed in one device.

In this paper we study quantum fluctuations of charge of a normal grain connected by tunnel junction to a superconducting lead. We show that due to the existence of the gap Δ in the lead the steps of the Coulomb staircase remain vertical at any G. The fluctuations result in a finite slope of the plateaus of the staircase. Unlike the N-I-N case, the slope may remain small even at $G \gg e^2/h$. One can suppress the gap by applying a magnetic field,[7] thus achieving a crossover between the limits of weak and strong charge fluctuations in one sample. The main result of the paper is illustrated by Fig. 1, obtained under the assumption $\Delta \gg E_C$, where $E_C = e^2/2C$ is the charging energy of the electron box, and C is its total capacitance.

To account for the Coulomb interactions, we include the charging energy term $(\hat{Q} - q)^2/2C$ into the Hamiltonian $\hat{H}$ of the system. The external charge q here is proportional to the gate voltage. Then by statistical averaging of the obvious relation $\partial\hat{H}/\partial q = (q - \hat{Q})/C$, we find the average charge of the grain

$$Q(q) \equiv \langle Q\rangle = q - C\frac{\partial F(q)}{\partial q}. \tag{1}$$

Here $F(q) = -T\ln Z(q)$ is the free energy.

To find the partition function $Z(q)$, we use the standard effective action approach by Ambegaokar, Eckern and Schön.[12] In this formalism an auxiliary Hubbard-Stratonovich field $\varphi(\tau)$ is introduced to replace the quadratic in $\hat{Q}$ interaction term by a linear one. The electronic degrees of freedom are then traced out, resulting in the following expression for the partition function:

$$Z(q) = \sum_{m=-\infty}^{+\infty} Z_m e^{i2\pi mq/e}, \tag{2}$$

with

$$Z_m = \int_{\varphi(0)=0}^{\varphi(\beta)=2\pi m} D\varphi(\tau)\exp\left\{-\int_0^\beta \frac{C\dot{\varphi}^2}{2e^2}d\tau - S_t[\varphi]\right\}. \tag{3}$$

Here the summation over winding numbers m accounts for the discreteness of charge,[13] and β is inverse temperature. The tunneling contribution $S_t[\varphi]$ to the effective

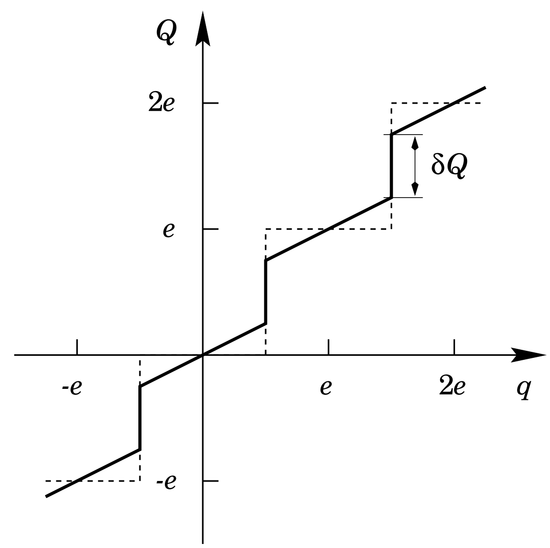

FIG. 1. Coulomb staircase for a normal grain connected to a superconducting lead. The steps remain vertical at any conductance. The plot corresponds to the case $\Delta \gg E_C$, and $G = (e^2/\hbar)\Delta/E_C$, see Eq. (7). Note that $G \gg e^2/\hbar$.

action was evaluated for a wide junction in Ref. 12. It is non-local in time τ; however, at large $\Delta \gg E_C$ its contribution reduces to a mere renormalization of capacitance C in the action in Eq. (3). The magnitude of the renormalization was found in Ref. 14. In the case of N-I-S junction the renormalized capacitance is

$$\tilde{C} = C + \hbar G/2\Delta. \tag{4}$$

In this approximation the action in Eq. (3) is quadratic, and we can easily find the dependence of Z_m on the winding number:

$$Z_m = Z_0 \exp\left[-\frac{\tilde{C}}{2e^2}\frac{(2\pi m)^2}{\beta}\right]. \tag{5}$$

To study $Z(q)$ at low temperatures $T \ll e^2/\tilde{C}$, it is convenient to transform the partition function (2) with the help of the Poisson summation formula. Equations (2) and (5) then yield

$$Z(q) = Z_0\sqrt{\frac{e^2}{2\pi\tilde{C}T}}\sum_{n=-\infty}^{+\infty}\exp\left[-\frac{(ne-q)^2}{2\tilde{C}T}\right]. \tag{6}$$

The largest term in sum (6) corresponds to an integer n which is closest to q/e. At $T \to 0$ this term dominates the sum, and using Eqs. (1) and (4), we find

$$Q(q) = e\,\mathrm{Int}\left(\frac{q}{e}+\frac{1}{2}\right) + \frac{q - e\,\mathrm{Int}\left(\frac{q}{e}+\frac{1}{2}\right)}{1+\frac{\Delta}{E_C}\frac{e^2}{\hbar G}}, \tag{7}$$

where $\mathrm{Int}(x)$ is the integer part of x. This is the central result of the paper; see also Fig. 1. At $G \to 0$ the second term in Eq. (7) vanishes, leading to a perfect staircase. At non-zero G the second term describes the finite slope of the plateaus. According to Eq. (7) the steps in $Q(q)$ remain sharp at any conductance, in contrast to the case of a normal lead. Even at $G \sim e^2/\hbar$ the heights of the steps $e/(1 + E_C\hbar G/\Delta e^2)$ are still close to their nominal value e.

The fact that $Q(q)$ is discontinuous at half-integer q/e is due to the density of states in the superconductor vanishing at the Fermi level, and therefore does not rely on the assumption $\Delta \gg E_C$. Indeed, it is known[3] that the problem of charge fluctuations in an electron box can be mapped onto an effective spin-$\frac{1}{2}$ Kondo problem. It follows from Eq. (30) of Ref. 3 that the height of the charge step is $\delta Q = 2e\mu$, where μ is the renormalized value of the spin of the Kondo impurity. In the case of a normal metal the spin is completely screened, $\mu = 0$, and the steps are smeared completely, $\delta Q = 0$. If at least one of the leads is a superconductor, the spin μ is not comletely screened, and the step height δQ remains finite.

At finite but low temperature $T \ll e^2/\tilde{C}$, the steps are smeared slightly. To account for the smearing, one needs to retain two terms, with $n = \mathrm{Int}(q/e)$ and $n = \mathrm{Int}(q/e) + 1$, in the sum (6). In this approximation the charge becomes

$$Q(q) = e\,\mathrm{Int}(q/e) + \frac{\tilde{C}-C}{\tilde{C}}[q - e\,\mathrm{Int}(q/e)] + e\,\frac{C}{\tilde{C}}\left[1+\exp\left(\frac{e^2[\mathrm{Int}(\frac{q}{e})+\frac{1}{2}-\frac{q}{e}]}{\tilde{C}T}\right)\right]^{-1}. \tag{8}$$

One can easily check that in the limit of zero temperature this result reproduces Eq. (7).

The effective action technique used here is applicable for tunnel junctions of wide area, where the conductance is distributed over a large number $N \gg 1$ of transverse modes. At finite N one may have to take into account the possibility of two-electron (Andreev) tunneling. Although the steps in the dependence $Q(q)$ remain vertical, the plateau conductance due to such processes acquires a correction, which can be estimated as $\delta Q \sim (\hbar G/e^2)^2 N^{-1}$. The diffusion of electrons inside the grain may enhance this correction. The result can be expressed in terms of effective number of channels[15] $N_{\rm eff} \lesssim N$, which still remains very large due to the smallness of the Fermi wavelength in metals.

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